On the Two-sided Market Dynamics in the Diffusion of Electric Vehicles

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***Abstract*—A two-sided market model is developed for the diffusion of electric vehicles and charging facilities. The model includes the consumers on the one side and investors of charging facilities on the other. The consumers' decisions are either to choose a type of vehicles (electric vehicle(EV) or gasoline vehicle(GV)) or to wait for more tractable options. Among the factors that influence their decisions are the price of charging service and the availability of charging stations. The investors, on the other hand, decide whether to invest in additional charging facilities and how to operate them after the facilities are built. Their decisions are based on the expected profit of investments or delay investments at a more profitable time. The dynamics of the number of EVs in the market is therefore intertwined with that of the number of charging stations, and they are modeled as a discrete time Markov decision process. An analytic and simulation studies on the equilibrium of the two-sided market dynamics is presented.**

***Index Terms*—Two-Sided Market; PHEV; Price Equilibrium; Social Welfare**

I. Introduction

The market share of electric vehicles (EV) has grown steadily in recent years, increasing almost 800% since 2011. Despite of the growth, the overall EV market share remains less than 1% as in July 2014. The reason behind the growth of EV, or the lack of it, is multifaceted. The growth is driven in part by the increasing awareness of environmental impacts of fossil fuel vehicles, the superior design and performance of some EVs, and, by no small measure, the tax credit provided by the federal and state governments. On the other hand, the EV industry still faces strong skepticism due to the high cost of EV, the limited driving range, and the lack of adequate public charging facilities.

A similar trend exists in the deployment of public charging facilities. Since the first quarter of 2011, the number of public charging stations in US has grown 700% by the end of 2013, due in part to the direct and indirect investments of federal and local governments. The Department of Energy (DoE) of the United States, for example, has provided $230 in 2013 to establish 13,000 charging stations [1] It is hoped that such investments will stimulate the EV market, driving its market share on a path toward long term growth and stability. The growth trends of EV and EV charging station (EVCS) have strong temporal and geographical couplings. This is due to the so-called two-sided market effects; the growth of EV attracts investments on EVCS, and the increasing presence of EVCSs makes EV more attractive to consumers. Similarly, the lack of EVCSs limits the growth of EV market share, which in turns inhibits new investments essential to the healthy growth and stability of the EV market.

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This paper focuses on the interactions between the two sides of EV-EVCS markets: the EV consumer on the one side and the investor of EVCS on the other. In particular, we formulate a sequential game model for the two-sided EV-EVCS market, which allows us to address analytically and numerically some of the following issues: how does the consumers decision of EV purchase interacting with that of the investor of EVCS facilities? How is the EV market share affected by the price of EV, the cost of EV charging, and the size of EVCS market? How does the EVCS investor maximize its profit by choosing sites of EVCS from a list of candidate locations? Are there differences between the market solution to EVCS investment and that by a social planner?

A. Summary of results

The main contribution of this work is an analytical study about the indirect network effects between the EV consumer and the EVCS investor. To this end, we introduce a complete Stackelberg game model for the two-sided EV-EVCS market with the investor as the leader and the consumer the follower. Through profit maximization, the investor decides whether to build CSs chosen (optimally) from a list of candidate CS sites or defer its investment. The candidate CS sites are heterogeneous; each CS site may have different favorable rating and different operation and building costs. The consumer, on the other hand, observes investors decision that defines the location of CSs and the cost of charging and decides whether to purchase an EV or a gasoline alternative.

We provide the solution of the Stackleberg game that includes the optimal decision for the consumer and the investor. Under a random utility maximization (RUM) model of the consumer, we show that the optimal policy is a threshold policy on the consumer preference. The closed form expression for the decision threshold t^* is obtained which is a function of, among others, the price of EV and the investors decision on the number/location of charging stations and the charging prices at those locations. The optimal decision threshold of purchasing an EV gives directly the EV market share as $\eta = 1 - t^*$, from which we examine how the investors decision and EV price affect the overall EV market share.

We first obtain the optimal operation decision by the investor by fixing the set of CS sites to build. We show that the optimal pricing for EV charging at these sites is such that profits generated from these sites are equal. We show further that the optimal pricing converges to a constant mark-up of the operating cost as the number of EV charging sites increases, which is the result of the monopolistic competition of EVCS market.

The optimal decision in choosing which CS site to build (or defer investment) is more complicated and is combinatorial in nature. We provide a greedy heuristic and show that the heuristic is asymptotically optimal as the number of CS sites to build increase.

Finally, we examine the difference between the social welfare optimizing solution and that of the market solution. We show that, when the number of charging stations is large, the market solution gives smaller number of CS sites than that from social welfare optimization.

B. Related work

There is an extensive literature on the two-sided market and cross network effects for various products; see *e.g.,* [2] on the CD player and CD title market, the video console and video game market [3], [4], [5], the hardware and software market [6], and the yellow page and advertisement market [7]. Rochet and Tirole in [8] proposed a restrictive definition of two-sided market. Caillaud and Jullien pointed out in [9] that, one side of the market always waits for the action from the other side. So it is critical for players to take right move, especially when the platform launches. [10] considered the market of credit cards and analyzed the competition strategy. The work of Li *et al.* [11] and the current paper represent the first analyzing the two-sided EV and EVCS market and related indirect network effects. The work in [11] focuses on the empirical study of indirect network effects whereas the current paper focuses on the theoretical analysis.

There is a growing literature on the EVCS investment from the operation research and engineering perspectives. For example, the charging station deployment has been formulated as an optimization problem from the social planner's point of view in [12], [13], [14]. A location competition of charging stations is considered in [15], where the discrete decision model are used in consumers' choice.

C. Organization

This paper is organized as follows: the structure of the two-sided market and a Stackleberg game model are described in Sec. II. The solution of the game is obtained through a backward induction. In Sec. III, the consumers' model and the optimal decision are stated. The investor's model and optimal strategy are presented in Sec. IV. In Sec. V, we consider social welfare optimization in choosing sites of EVCS investment. Sec. VI concludes the paper.

D. Summary of results

This paper is an first analytical study about the interactions between the two sides of the EV market: the EV and the EV charging services (EVCS). On modeling, the consumers' surplus from charging service and purchasing vehicles are formulated as a linear combination of utilities, prior preference, and prices. The consumer discrete choice model leads to a mutinormial logistic model of market share of charging stations and a threshold policy in vehicle decisions. These results give a explicit form of EV market and the relationship between the market share and the scale of EVCS. Our study shows the successful launch of EV not only depends on the characteristic of EV but also highly relies on the development of charging stations.

On the EVCS investor's decision, an asymptotic optimal strategy of building stations are proposed under the monopoly assumption. First, we formulate a constrained optimization where the profit of building charging stations is maximized. The optimization is subject to budget constraint with given location candidates set. Second, the optimal charging price structure is derived, which yield to a uniform profit over different charging stations. Then a simple ranking heuristic algorithm is proposed for station location selection and the asymptotic optimality is shown, which significantly reduces computation complexity from exhaustive searching for all possible combination.

In the end, the social welfare optimization is considered. The result suggests at the social welfare optimal point, more charging service is needed than the investor optimal point. The relationship between the EV market share and the size of EVCS justifies the subsidy policy to both EV purchase and station building.

E. Related work

The EV and charging station problem is considered in this paper under the setting of two-sided market, which is usually used to study the indirect network effect between two parties interact through a "platform". In a two-sided market, the "platform" connects two sides of agents (typically consumers and producers) and the decisions of each side affect the outcome of the other. A typical example of two-sided market is the smart-phone. The smart-phones with different operation system(OS), such as iPhone and Android phone, connect consumers and software companies. Consumers observe the characteristics of phones and APPs attached to each OS and make their purchase decision. Meanwhile, the software company choose which OS to write APPs for based on the population of users.

There are increasing literatures on indirect network effect and two-sided market. Rochet and Tirole in [8] proposed a restrictive definition of two-sided market. Caillaud and Jullien pointed out in [9] that, one side of the market always waits for the action from the other side. So it is critical for players to take right move, especially when the platform launches. [10] considered the market of credit cards and analyzed the competition strategy. [16] estimated a discrete choice model of dynamic consumer demand in video game console two-sided market, where the consumer discrete choice model is similar to the model used in this work.

The moltunomial logit (MNL) model is used to model the consumer behavior, which is firstly introduced by McFadden in [17] to model the action of consumers facing discrete choices.

McFadden showed the extreme value type one distribution in consumer's preference leads to logistic probability in choice. Following that, the MNL model is widely used in discrete choice model [16], [18].

F. Organization

This paper is organized as follows: the structure of the two-sided market and the game are described in Sec. II. In Sec. III, the consumers' model and the optimal decision are stated. The investor's model and optimal strategy are presented in Sec. IV. The social welfare of the market is discussed in Sec. V and Sec. VI concludes the paper.

II. Two-sided Market Model and the Structure of the Game

We formulate the two-sided market as a two-player Stackelberg (sequential) game with complete information. The players are the EVCS investor as the leader and the EV consumer the follower. We define the investor and consumer models separately next.

A. The investor

Let $\bar{\mathcal{C}} = \{s_i = (f_i, c_i), i = 1, \cdots, N_L\}$ be the set of potential sites for charging stations known to the investorwhere f_i is the favorability rating and c_i the marginal operation cost.* Given the candidates set $\bar{\mathcal{C}}$ and the utility function of the consumers, the investors' action space is defined as $\{\mathcal{R}^{N_L} \times \mathcal{R}^{N_L} : \mathcal{C} \times \vec{\rho}\}$, where $\mathcal{C} \subseteq \bar{\mathcal{C}}$ is the set of charging stations selected to be built and $\vec{\rho} = (\rho_1, \cdots, \rho_{|\mathcal{C}|}) \in \mathcal{R}^{|\mathcal{C}|}$ is the charging prices vector. The investor predicts the action of consumers and by choosing $\{\mathcal{C} \times \vec{\rho}\}$ to maximizes the investment profit within the budget B. The investment optimization is stated as

$$\begin{array}{ll} \max_{\mathcal{C},\vec{\rho}} & \Pi(\mathcal{C}, \vec{\rho}) - \sum_{i=1}^{|\mathcal{C}|} F(s_i) \\ \text{such that} & \sum_{i=1}^{|\mathcal{C}|} F(s_i) \leq B \end{array} \tag{1}$$

where Π is the charging profit collected from consumers, $F(s_i)$ is the building cost of station i.

B. The consumer

Observing the charging stations and charging prices, $\{\mathcal{C}, \vec{\rho}\}$, the consumers optimally make the vehicle purchase and the charging choice. The action space of consumers are defined as $\{V, j\}$, where $V \in \{E, G\}$ is the vehicle choice from EV and GV; $\{j \in \{1, \cdots, N_E(N_G)\}\}$ is the charging(gas) station choice. The consumers optimally choose $\{V, j\}$ to maximize the vehicle surplus and charging(refueling) surplus.

The consumers surplus model of purchasing a vehicle is assumed as follows:

$$\begin{array}{l} V_E = \beta \mathbb{E} U_E - p_E + \Phi + \epsilon_E \\ V_G = \beta \mathbb{E} U_G - p_G + \Phi + \epsilon_G \end{array} \tag{2}$$

where

- $\mathbb{E}U_E(\mathbb{E}U_G)$ is the expected maximum charging (refueling) utility;
- $p_E(p_G)$ is the vehicle price of EV(GV);
- Φ is the utility of owing a vehicle;
- $\epsilon_E(\epsilon_G)$ is the prior preference of vehicles;

The decision of a consumer on vehicles is to given by:

$$\max_{V \in \{E,G\}} \{V_E, V_G\} \tag{3}$$

After vehicle purchase, the EV owner need to optimally select charging stations to maximize the charging surplus. The consumer surplus at station i is assumed as:

$$U_i = \alpha f_i - \rho_i + \epsilon_i, i = 0, \ldots, N_E \tag{4}$$

where

- f_i the favorability rating;
- ρ_i the charging price;
- ϵ_i the prior preference of stations;
- $i = 0$ indicates charge at home.

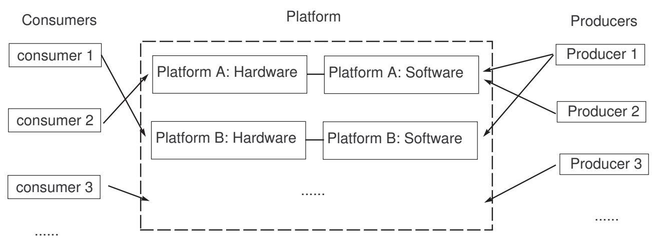
Given the realization of $\vec{\epsilon} = (\epsilon_0, \ldots, \epsilon_{N_E})^T$, the EV owner chooses charging station $j (j \in \{0, 1, \ldots, N_E\})$ to maximize his charging utility. The maximum charging utility can be stated as

$$U_E(\vec{\epsilon}) \triangleq U_j(\vec{\epsilon}) = \max_{i \in \{0, \ldots, N_E\}} U_i(\vec{\epsilon}) \tag{5}$$

It is assumed that there are already enough gas stations such that the investor will not consider to build new gas stations. Instead, the investor will put the rest of money in the bank and earn a interest at rate γ. Thus, the utility of fueling will not change as the decisions of investor change. So we can treat the maximized fueling utility, U_G, as a constant.

C. Game Description

The structure of the two-sided EV Market can be illustrated as following:



- Investors' Stage:
  - Given locations $\bar{\mathcal{C}}$, determine $\mathcal{C}$, or just put the money in the bank;
  - After building $(N_E \triangleq |\mathcal{C}|)$ charging stations, the investor determines the charging price ρ_i.
- Consumers' Stage:
  - Observing $\{\mathcal{C}, \vec{\rho}\}$, determine $V \in \{E, G\}$;
  - After purchased vehicles, choose charging station $j \in \{0, \cdots, N_E\}$ to charge .

*The marginal cost ($/mile) here is the locational marginal price of wholesale electricity ($/kWh) normalized by EV efficiency (miles/kWh).

During this game, we are interested in the optimal strategy of both the consumers and the investor. To answer these questions, we will analysis the game backwards. We first consider the charging utility model and charging decisions of consumers. After that, the consumers' decision on vehicles and market share of EV are discussed. In Sec. IV, the optimal charging prices and strategy of building charging stations are presented.

III. Consumer Decisions

In this section, the optimal decisions of consumers are presented. The optimal decision on charging station is simply pick the one with maximum surplus. The decision on vehicle is in a threshold form. As a result, the market share of the charging station is a multinomial logit model.

A. Assumptions

Before we look into the decisions of consumers, let us summarize the assumptions of consumers as follows:

- Consumers are I.I.D..
- Number of consumers is normalized to 1.
- The average charging demand is normalized to 1.
- The prior preference of station i, ϵ_i, is i.i.d and follows the extreme value type one distribution with the pdf:

$$f(\epsilon) = e^{-\epsilon} e^{-e^{-\epsilon}}$$

- The prior preference of vehicle, $\epsilon_E(\epsilon_G)$, follows the uniform distribution:

$$\begin{array}{l} \epsilon_E = \phi t_E, t_E \sim \mathcal{U}(0,1) \\ \epsilon_G = \phi t_G, t_G = 1 - t_E \end{array} \quad (6)$$

The extreme value type one distribution model is widely used in the discrete choice model in two-sided market. McFadden first introduced the extreme value distribution in the discrete choice model and showed it leads to the multinomial logit model. By assuming the vehicle preference follows the uniform distribution, we assume the consumers are lying on a unit line.

B. Consumer Decision and EV Market Share

By assuming the preference of stations the extreme value type one distribution, we can derive the expected maximized charging utility of consumers as follows:

$$\begin{aligned} \mathbb{E}U_E &= \int U_E(\vec{\epsilon}) f(\vec{\epsilon}) d\vec{\epsilon} \\ &= \ln\left(\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k)\right) \\ &\triangleq \ln\left(\sum_{k=0}^{N_E} q_k\right) = \ln(q) \end{aligned} \quad (7)$$

where $\vec{\epsilon} = (\epsilon_0, \cdots, \epsilon_{N_E})$ is the preference vector; $q_k = \exp(\alpha f_k - \rho_k)$ is the exponential utility of the kth station.

Clearly the expected maximized charging utility, $\mathbb{E}U_E$, is increasing in f_i and N_E and decreasing in ρ_i, which indicates more attractive charging stations with cheaper charging price benefit consumers more. $\mathbb{E}U_E$ is also an concave function, which implies the marginal utility of new charging stations is decreasing. Note the consumer vehicle surplus $\max\{V_E, V_G\}$ is non-decreasing in $\mathbb{E}U_E$, which implies more charging stations increase the overall consumers surplus.

Assuming the consumers' preference of vehicles lying on a unit line, we can derive consumers' optimal decision on vehicles. A type t_E consumer purchases EV if

$$V_E(\mathcal{C}, \vec{\rho}, t_E) \geq V_G(t_G)$$

which indicates

$$t_E \geq -\frac{\beta \ln(\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k)) - p_E}{2\phi} + \frac{\beta U_G - p_G}{2\phi} + \frac{1}{2}$$

Denote the right hand side as t^*. Provided $0 \leq t^* \leq 1$, all consumers lying in $[t^*, 1]$ will purchase EV, and other consumers will purchase gas vehicles. Indeed, t^* is the location of the indifferent consumers such that the vehicle utilities of EV and gas vehicle are the same. Thus we have the following theorem:

Theorem 1 (Consumer choice): The optimal consumer decision is a threshold policy on the consumer preference $t_E \sim \mathcal{U}(0,1)$

$$\begin{cases} t_E \geq t^* & \text{purchase electric vehicle} \\ t_E < t^* & \text{purchase gasoline vehicle} \end{cases}$$

where

$$t^* = \left[\frac{\beta U_G - p_G}{2\phi} + \frac{1}{2} - \frac{\beta \ln(\sum_{i=0}^{N_E} \exp(\alpha f_i - \rho_i)) - p_E}{2\phi}\right]_0^1$$

The EV market share for the optimal consumer choice is given by $\eta = (1 - t^*)$.
The charging station market share is given by

$$P_i = \frac{\exp(\alpha f_i - \rho_i)}{\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k)} \triangleq \frac{q_i}{q}$$

The EV market share η is increasing in the number of charging station N_E and favorability rating f_i, decreasing in EV price p_E and and charging price ρ_i, provided $0 \leq \eta \leq 1$. More attractive charging stations and cheaper prices motivate more use of EV. This justifies the tax credit subsidy to EV purchase and the charging station. The market share of charging station i, P_i, is indeed the probability that the charging surplus at i, U_i, is the maximum under the extreme value type one distribution assumption.

The EV market share versus number of charging stations with different EV prices is plotted in Fig. 1. When the EV price is lower, the critical number of charging stations to make EV has a non-zero market share is smaller. With the same number of stations, cheaper EV price also accelerates the market share faster.

The critical number of charging stations versus the charging price ρ_i is plotted in Fig. 2. The critical number increases exponentially in charging price, which indicates the lower charging price is one key to help EV launch successfully.

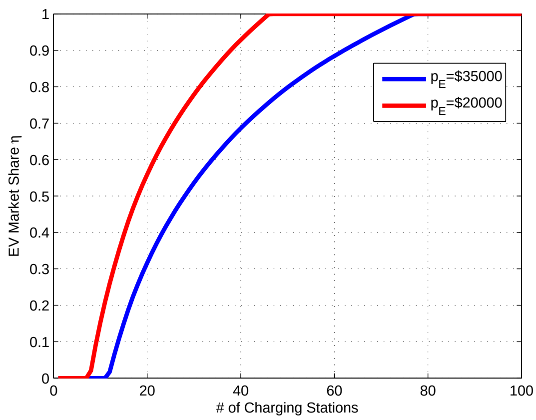


Fig. 1: EV Market Share Vs. # of Charging Station

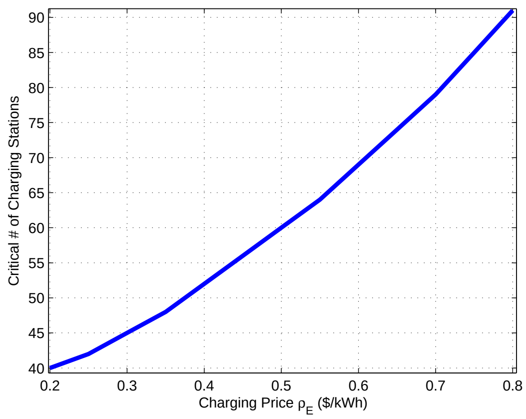


Fig. 2: Critical # of stations Vs. charging price ρ_i

IV. Investor Decisions

In this section, the investor model on charging prices and station building will be analyzed. The structure of the optimal charing price is presented. A heuristic algorithm of choosing charging station set $\mathcal{C}$ is proposed, which is asymptotically optimal.

A. Investor Decision Model and Assumptions

1) Investor Model: Theorem 1 gives the market share of charging station i as P_i. Thus given the charging station set $\mathcal{C} = \{s_i = (f_i, c_i), i = 1, \cdots, N_E\}$, the profit of charging station i can be stated as

$$\Pi_i = \eta(\mathcal{C}, \vec{\rho}) P_i(\mathcal{C}, \vec{\rho})(\rho_i - c_i)$$

where

- $\eta(\mathcal{C}, \vec{\rho})$ the fraction of consumers who own EV;
- $P_i(\mathcal{C}, \vec{\rho})$ the market share of station i;
- c_i the marginal operation cost of station i;

The total profit collected from charging is given by

$$\Pi = \sum_{i=1}^{N_E} \Pi_i = \eta(\mathcal{C}, \vec{\rho}) \sum_{i=1}^{N_E} P_i(\mathcal{C}, \vec{\rho})(\rho_i - c_i)$$

2) Assumptions: Here the assumptions about the investor are summarized:

- There is only one investor. There is no competition among the charging stations.
- The building cost of a charging station is a constant, $(1+\gamma)F_0$, where γ is the interest rate of bank.
- The investor knows the utility function of consumers.

Since the game of investor is assumed to be monopoly, the investor can control all the charging prices to maximize his profit. The building cost is assumed constant and independent from the location choice s_i. The investor knows the utility function of consumers and can predict their decisions. By taking this advantage, the investor can optimally choose the location set $\mathcal{C}$ and the charging price.

B. Investor Decision

For the decisions of investor, we also apply backward analysis. Firstly, assuming the number and locations of stations are given, we derive the optimal charging price. Following that, we discuss the optimal strategy to choose locations given the station number. Then the optimal invest strategy is completed by analysis on how many stations to build.

1) Charging Price: Assume the location $\mathcal{C}$ is given, the investor determines the optimal charging price $\vec{\rho}$ to maximize the total profit

$$\max_{\vec{\rho}} \Pi = \max_{\vec{\rho}} \eta(\vec{\rho}) \sum_{i=1}^{N_E} P_i(\vec{\rho})(\rho_i - c_i)$$

Solving the optimization problem, the optimal charging price ρ_i^* is given by

Theorem 2 (optimal charging price): For fixed set of charging stations $\mathcal{C} = \{(f_i, c_i), i = 1, \cdots, N_E\}$, the optimal charging price ρ_i^* generates uniform profit across charging stations. In particular,

$$\rho_i^* - c_i = \frac{1}{\frac{\beta(1-P_0(\vec{\rho}^*))}{2\phi\eta(\vec{\rho}^*)} + P_0(\vec{\rho}^*)} \tag{8}$$

where $P_0(\vec{\rho}^*) = \frac{\exp(\alpha f_0 - \rho_0^*)}{\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k^*)}$ is the probability consumers charge at home and ρ_0^* is the cost charging at home.

Note the right hand side of (8) is the same for any i. The profit of each station from single consumer is the same. Since $P_0 \geq 0$, the revenue is strict positive. As $N_E \to \infty$, the EV market share $\eta \to 1$ and $P_0 \to 0$, we have the limit of the revenue as follows:

Remark 1:

$$\rho_i^* - c_i \to \frac{2\phi}{\beta}$$

as $N_E \to \infty$.

When consumers care more about the charging utility and less about the vehicle itself, which indicates β is large while ϕ is small, the investor need to lower the price and charging stations earn less profit from each consumer.

2) Strategy of building stations: After obtaining the optimal charging price, the investor need to decide the set of charging locations to invest. The optimal investment problem is stated as: given the location candidates $\bar{\mathcal{C}} = \{s_i = (f_i, c_i),\ i = 1, \cdots, N_L\}$,

$$\max_{\mathcal{C} \subseteq \bar{\mathcal{C}}} \quad \Big\{ \Pi(\mathcal{C}, \vec{\rho}^*) - \sum_{i=1}^{|\mathcal{C}|} F(s_i) = \Pi(\mathcal{C}, \vec{\rho}^*) - (1+\gamma) N_E F_0$$
$$\text{such that} \quad N_E F_0 \leq B$$

Clearly, the optimal charging price is a function of station number N_E and station set $\mathcal{C}$. But the lack of close form of ρ_i^* brings difficulty in discussing building strategy. But the profit converges to a constant and charging station number increases. Based on this, a heuristic greedy algorithm is proposed in Algorithm 1.

Algorithm 1 Greedy Investment Algorithm

1. Compute the exponential utilities $v_i = \exp(\alpha f_i - c_i)$ and sorted list $\{v_{(i)}\}$;
2. Set $N = 1$;

while $N \leq N_L$ **do**
  Compute $\tilde{P}_N \triangleq \Pi(c_1, \cdots, c_N) - \sum_{i=1}^{N} F_i(s_i)$;
  if $\tilde{P}_N < \tilde{P}_{N-1}$ or $\sum_{i=1}^{N} F_i(s_i) \geq B$ **then**
    STOP;
  else
    $N \leftarrow (N+1)$;
  end if
end while

In principle, the optimal investment decision need to exhaustively search for all possible combination of charging locations and compare the profit. The fact of uniform profit converges to a constant makes it possible to separate the price decision and the location choice. Indeed, we have

Theorem 3 (Asymptotic optimality): If the cost of charging stations is constant, then the greedy algorithm is asymptotically optimal (as $N \to \infty$). ■

Proof: See Sec. VII.

The greedy algorithm suggests that the investor should choose first to build charging stations at more attractive locations such as residential community or work places.

If the maximized profit $(\Pi(\mathcal{C}, \vec{\rho}^*) - \sum_{i=1}^{|\mathcal{C}|} F(s_i))$ is positive, then investor will decide to invest. Otherwise, no investment will be made and the money will go to the bank. This requires the EV price p_E and building cost F_0 small enough, which justifies the subsidies to EV purchase and building stations.

3) Government Subsidize: The optimal strategies of station investment and vehicle choice have been established above. The results suggests it is needed to keep the price of EV and building cost of charging station cheap enough to ensure the successful launch of EV. An example of different government subsidize is discussed in this section.

In Fig. 3, there is neither subsidy to EV purchase nor to charging stations. The optimal charging station number of investor is 60 and the market share of EV, η, is smaller than 1.

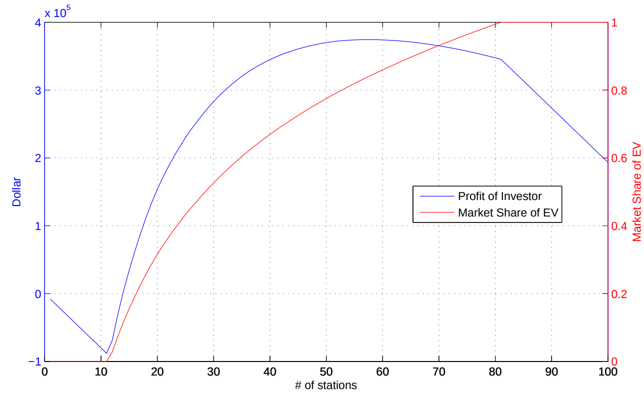


Fig. 3: No subsidize, $N_E^* = 60$, $\eta < 1$;

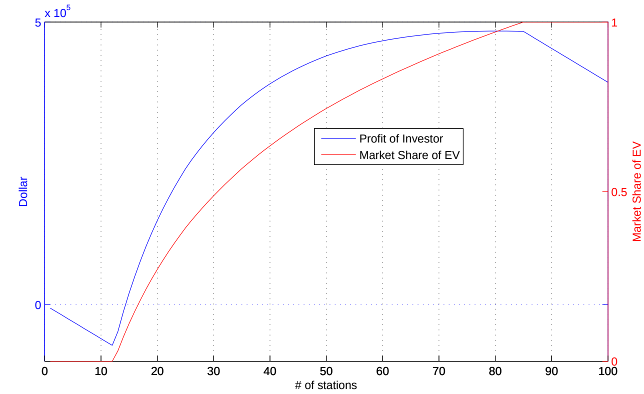


Fig. 4: Subsidy to Charging Stations, $N_E^* = 85$, $\eta = 1$;

If the government subsidizes charging stations, the investor is motivated to build more charging stations and more consumers tend to purchase EV with more charging facilities. In Fig. 4, each station receives a \$2000 subsidy and the optimal number of stations grows to 85, with EV market share equals to 1. The consumers' vehicle surplus increases because of more charging facilities. The subsidy deficits both sides of the market.

Instead, if the subsidy goes to the consumer side, more consumers will choose to own EVs and the market share of EV grows. Observing the population of EV, the investor is motivated to build more charging stations. In Fig. 5, instead of subsidizing investor, each EV purchase receives a \$6000 tax credit, which drives the EV market share to 1. The optimal number of charging stations also grows to $N_E^* = 65$. Note the investor does not need to build as many stations as the previous case because the market share of EV is driven by the government subsidy. The investor benefits from the government action as well.

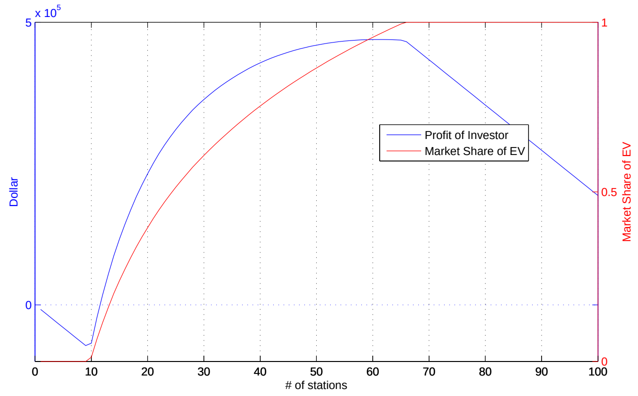


Fig. 5: Subsidy to Consumers, $N_E^* = 65$, $\eta = 1$;

V. Social Welfare

As a social planner, the government wants to maximize the social welfare, which is the sum of consumers' and investor's surplus. Denote the consumers' surplus by S_c, and the investor's by S_I. We have

$$\begin{aligned} S_C(\mathcal{C}, \vec{\rho}^*) &= \mathbb{E}(\max\{V_E(\mathcal{C}, \vec{\rho}^*, t_E), V_G(t_G)\}) \\ S_I(\mathcal{C}, \vec{\rho}^*) &= \Pi(\mathcal{C}, \vec{\rho}^*) - \textstyle\sum_{i=1}^{|\mathcal{C}|} F_i(c_i) \end{aligned}$$

where the consumers' surplus is the expected maximized vehicle surplus. The investor's surplus is the difference between the charging profit and the station building cost.

Assume the social planner can not determine the charging price or the vehicle price, it can only determine the set of charging stations. The investor's decision is stated as:

$$\max_{\mathcal{C} \subseteq \bar{\mathcal{C}}} S_C(\mathcal{C}, \vec{\rho}^*) + S_I(\mathcal{C}, \vec{\rho}^*)$$

The investor's surplus has been derived in Sec. IV. The consumers' surplus, S_C, can be stated as:

Lemma 1: Provided $0 \le \eta \le 1$, the consumers' surplus is

$$S_C(\mathcal{C}, \vec{\rho}^*) = [\phi(\eta((\mathcal{C}, \vec{\rho}^*))^2 + (\beta U_G - p_G) + \Phi - \frac{\phi}{2}]$$

It is shown in equation (8) that, the investor optimal charging price generates uniform profit across stations. When the number of charging stations is large, the profit can be approximated by a constant. Following a similar process, the social planner's strategy can be shown as also a ranking strategy. The station number of social optimizer is larger than the investor optimal number:

Theorem 4 (Social Welfare): Let $\mathcal{C}^*$ be the optimal set of charging stations determined by the investor, and $|\mathcal{C}^*| \gg 1$. Let $\mathcal{C}^{**}$ be the optimal charging locations determined by the social planner. Then $|\mathcal{C}^{**}| > |\mathcal{C}^*|$.

Proof: See Sec. VII. ■

When the investor wants to enter the charging station market, as the social planner, the government can set a minimum number of stations the investor needs to build to drive the charging station number to the social welfare optimizer. This fact justifies the regulation of Beijing government, requiring at least 18% of the parking spots need to have charger in all the new invested residential communities.

VI. Conclusion

The two-sided market of EV is considered in this work. A sequential Stackelberg game is formulated to analyze the indirect network effect between investor and consumers. The optimal operation decision of charging stations is shown as locational equal profit pricing. An asymptotic optimal algorithm of investment decision is proposed which reduces the computation complexity significantly. The social welfare optimization is discussed and it is shown that the social welfare optimizer requires more charging stations than investor optimizer.

These results give the relationship between the EV market share and the size of EVCS. This paper justifies the subsidy to EV purchase and charging station building, as well as the regulation on number of new built charging stations, are necessary to help EV launch successfully in the very beginning.

In this paper, the game is formulated as one shot. It will be interested to reformulate the market behavior as a repeated game. In the repeated game, the investor will observe the consequence of his action and adjust it in the next stage. Consumers' can also predict the trend of the charging stations and determine the optimal time to purchase vehicles. Based on the repeated game setting, a model fitting can be carried out using the sale data. After that, a set of quantified questions can be answered, such as what is the optimal way to spend the government budget to help EV market.

VII. Appendix

A. Proof of Theorem 3

As $N_E \to \infty$, $(\rho_i^* - s_i)$ converges to a constant. So in the analysis of building strategy, the optimal charging price is approximated by $\rho_i^* \approx c_i + \frac{2\phi}{\beta}$.

Firstly, fixing the number of stations to built as N_E, let us examine where to build these stations. Denote the exponential utility from station i as $q_i = \exp(\alpha f_i - \rho_i^*) \approx \exp(\alpha f_i - (c_i + \frac{2\phi}{\beta_1}))$ and the sum of utility as $q = \sum_{i=0}^{N_E} q_i$. The charging profit of the investor can be re-written as

$$\begin{aligned} \Pi(q) &= \eta(q) \textstyle\sum_{i=1}^{N_E} P_i(q_i, q)(\rho_i^* - c_i) \\ &= \frac{2\phi}{\beta_1}\eta(q) \textstyle\sum_{i=1}^{N_E} \frac{q_i}{q} \end{aligned} \quad (9)$$

Take partial derivative of the revenue with respect to q_i, we have

Lemma 2: The revenue of the investor is strictly increasing in q_i:

$$\frac{\partial \Pi(q)}{\partial q_i} > 0$$

Lemma 2 implies that, if given two station candidates j and k, fixing the other $(N_E - 1)$ stations, the one with larger $q_i = \exp(\alpha f_i - (c_i + \frac{2\phi}{\beta})), i \in \{j, k\}$ should be built. So we have the optimal strategy about where to build stations:

Lemma 3: Fixing the number of stations to built as N_E, the optimal strategy of building is to pick N_E candidates with largest $v_i = \exp(\alpha f_i - c_i)$ to build.
Lemma 3 shows the location decision of stations need to include both of the operation cost c_i and the favorability rating f_i. When the operation cost c_i are the same, the locations where consumers visit more should be given priority. This justifies the charging stations should be built at the attractive locations such as work places and the residential communities.

Before the optimal number of charging stations is considered, we can first sort the N_L candidate locations by q_i. Now instead of N_E, we can present the cost $F(N_E) \triangleq (1+\gamma)F_0 N_E$ as a function of $q = \sum_{i=0}^{N_E} q_i$. Since $q_i \geq q_{i+1}$, the cost $F(N_E(q))$ is a piece wise linear concave function of q. The partial derivative is piece wise constant and increasing in q. And we know the revenue $\Pi(q)$ is increasing in q. By looking into the second order derivative, we have

Lemma 4: As q increases, $\Pi(q)$ is first a convex function, then a concave function.

The curve of $\Pi(q)$, $F(q)$ and the derivative are plotted as follows:

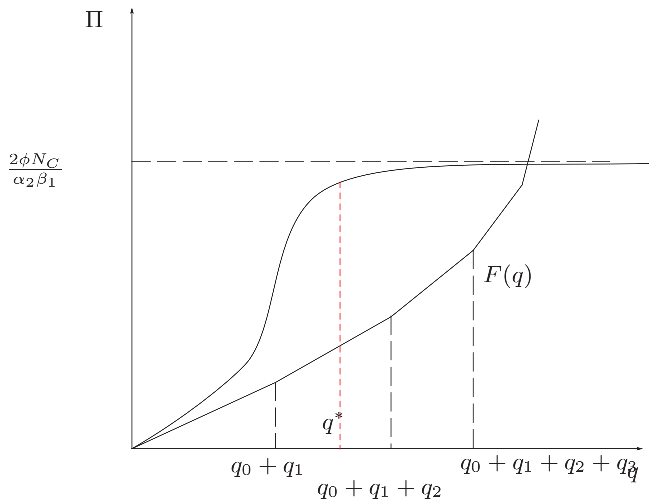


Fig. 6: Profit and Cost of stations

In Fig. 6, q^* is the optimal point to maximize the profit $(\Pi(q) - F(q))$. Fig. 7 shows the derivative of $F(q)$ is increasing and the marginal profit $\frac{\partial \Pi(q)}{\partial q}$ is first increasing then decreasing. There are at most two cross points in the derivative and the latter one is the optimal point. Combining Lemma 2, 3, 4, we have the asymptotic optimality.

To make building charging stations attractive $(\Pi(q^*) - F(q^*) > 0)$, the building cost F_0 and the EV price p_E need to be small enough. This justifies that the government need to subsidize the price of EV and the cost of building charging stations.

B. Proof of Lemma 4

Denote the sum of consumers' surplus and investor's revenue as $\bar{S}_W(q) = S_C(q) + \Pi(q)$, the social planner is

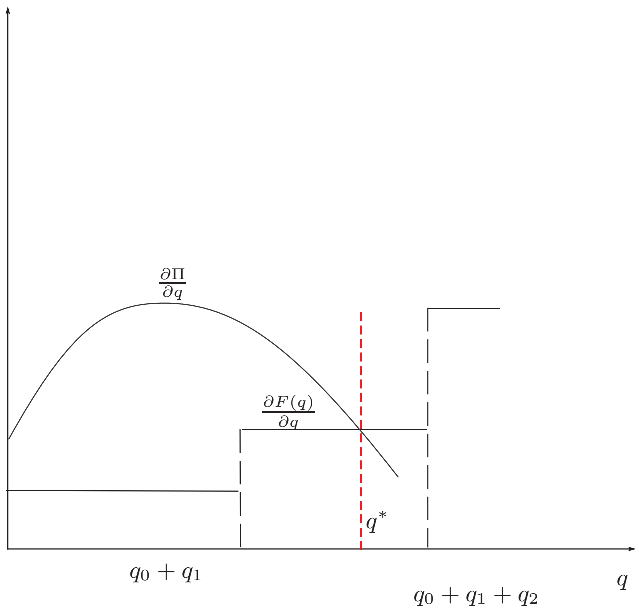


Fig. 7: Profit and Cost Derivative

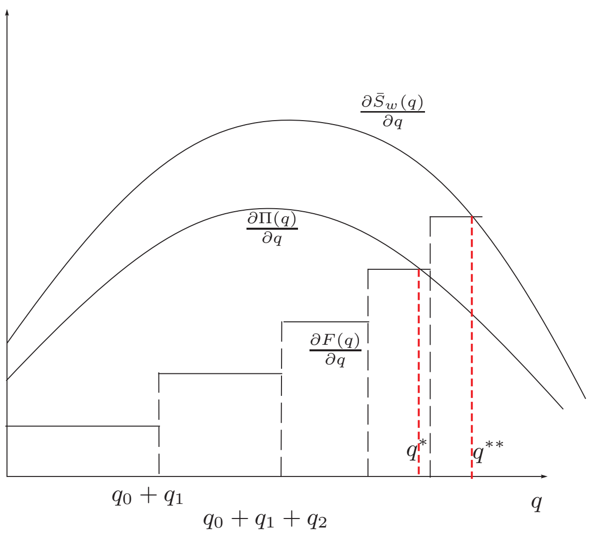


Fig. 8: Social Welfare

maximizing $(\bar{S}_W(q) - F(q))$.

Clearly, the consumers' surplus is increasing, $S_C(q)$, in q. So

$$\frac{\partial \bar{S}_W(q)}{\partial q} = \frac{\partial S_C(q)}{\partial q} + \frac{\partial \Pi(q)}{\partial q} > \frac{\partial \Pi(q)}{\partial q}$$

We plot the derivative of the social welfare as well as the investor surplus in Fig 8. Clearly, the optimal social welfare point q^{**} is also the cross point of $\frac{\partial F(q)}{\partial q}$ and $\frac{\partial \bar{S}_W(q)}{\partial q}$. Since $\frac{\partial \bar{S}_W(q)}{\partial q} > \frac{\partial \Pi(q)}{\partial q}$, it is always true that $q^{**} \geq q^*$, which implies the social optimal point requires more charging stations to build.

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